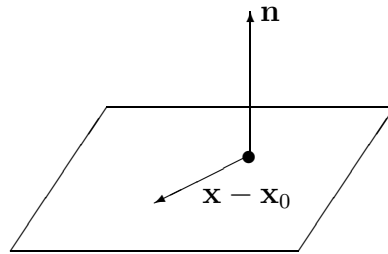


# QUESTION

- (a) Find a unit normal to the surface  $xyz + 3x^2 = 4$  at the point  $(1,1,1)$ .
- (b) Find the equation of the tangent plane to the surface  $3yz^2 + 2x^2 - 4xy^2 = 3$  at the point  $(0, 1, 1)$ .
- (c) Find the angle between the surfaces  $2x^2 + y^2 + z^2 = 4$  and  $x^2 + y^2 - z = 1$  at the point  $(1, 1, 1)$ .

# ANSWER

- (a)  $f = xyz + 3x^2 - 4$   $\mathbf{x}_0 = (1, 1, 1)$   
 $\nabla f = (yz + 6x, xz, xy)$   $\nabla f|_{\mathbf{x}_0} = (7, 1, 1)$   $\mathbf{n} = \pm \frac{1}{\sqrt{51}}(7, 1, 1)$
- (b)  $f = 3yz^2 + 2x^2 - 4xy^2 - 3$   $\mathbf{x}_0 = (0, 1, 1)$   
 $\nabla f = (4x - 4y^2, 3z^2 - 4x, 6yz)$   $\nabla f|_{\mathbf{x}_0} = (-4, 3, 6)$   
 Equation of tangent plane is  $(\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{n} = 0$



$$\begin{aligned} -4(x - 0) + 3(y - 1) + 6(z - 1) &= 0 \\ -4x + 3y + 6z &= 9 \end{aligned}$$

- (c)  $f_1 = 2x^2 + y^2 + z^2 - 4$   $f_2 = x^2 + y^2 - z - 1$   $\mathbf{x}_0 = (1, 1, 1)$   
 $\nabla f_1 = (4x, 2y, 2z)$   $\mathbf{n}_1 = \frac{1}{\sqrt{24}}(4, 2, 2) = \frac{1}{\sqrt{6}}(2, 1, 1)$   
 $\nabla f_2 = (2x, 2y, -z)$   $\mathbf{n}_2 = \frac{1}{3}(4, 2, 2)$   
 Angle between surfaces = angle between normals =  $\theta$   
 $\cos \theta = \mathbf{n}_1 \cdot \mathbf{n}_2 = \frac{1}{\sqrt{6}} \frac{1}{3} 5$   
 $\theta = \cos^{-1} \left( \frac{5}{3\sqrt{6}} \right)$